Effect of trapped ions and nonequilibrium electron-energy distribution function on dust-particle charging in gas discharges

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Dust-particles charging in a low-pressure glow discharge was investigated theoretically. The dust-particle charge was found on the basis of a developed self-consistent model taking into account the nonequilibrium character of electron distribution function and the formation of an ionic coat composed of bound or trapped ions around the dust particle. The dust-particle charge, the radial distributions of electron density, free and trapped ions densities, and the distribution of electrostatic potential were found. It was shown that the non-Maxwellian electron distribution function and collisional flux of trapped ions both reduce the dust-particle charge in comparison with that received with the help of the conventional orbital motion limited (OML) model. However, in rare collisional regimes in plasma when the collisional flux is negligible, the formation of ionic coat around a particle leads to a shielding of the proper charge of a dust particle. In low-pressure experiments, it is only possible to detect the effective charge of a dust particle that is equal to the difference between the proper charge of the particle and the charge of trapped ions. The calculated effective dust particle charge is in fairly good agreement with the experimental measurements of dust-particle charge dependence on gas pressure.

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I. INTRODUCTION

Dusty or complex plasma is an ionized gas of electrons, ions, and micron-sized particles or grains which are usually negatively charged (10^3-10^4 electronic charge). Dusty grains can be found either in space (e.g., planet rings, interstellar molecular clouds, cometary tails) or in different technological processes (e.g., plasma chemical deposition and coating, thermonuclear reactors, etc.). In laboratory conditions, dusty plasma is investigated in radio-frequency (RF) plasma and in direct-current (DC) glow discharges. Many interesting phenomena are observed and investigated in dusty plasma, e.g., formation of dusty structures (dust crystals, liquids and gases), phase transitions, wave propagations, and different kinetic processes. For the current state of the field, see recent review papers [1–6].

The dust particle charge is the most important parameter of dusty plasma. It determines the electrostatic interaction of the particle with other dust particles, ions, and electrons in surrounding plasma, and with external electric field. The knowledge of a grain charge is important for understanding different physical processes in dusty plasma. The negative charge on a grain embedded in a plasma background is determined by the balance of electron and ion fluxes to its surface. A number of experimental techniques have been developed to evaluate the charge on a dust grain immersed in plasma [7–17]. Most of the reported techniques used the method of levitation of a charged particle in plasma sheath or in striations of a DC glow discharge when electrostatic and gravitation forces acting on the grain get balanced. The measurements based on electrostatic interaction of two dusty particles were used in [15]. The dust-particle charge in a DC glow discharge in [16,17] was estimated from the balance of ion drag force and the action of the electric field on the charged particle. However, all these experimental results are very sensitive to the theoretical model used for the interpretation of a charged grain interaction in plasma with external electric field and other charged particles.

Theoretical attempts to solve the problem of charging of grain or small probe immersed in plasma were started by Langmuir [18] in the 1920s, followed by many authors [19–28]. The orbital motion limited (OML) theory for spherical grains in low density plasma is often applied to obtain the charge of dust particles [1,29,30]. This approach deals with collisionless electron and ion trajectories in the vicinity of a small probe or dust particle and only the conservation laws of energy and angular momentum to calculate electron and ion fluxes to the surface of grain are used. This would seem to be quite a reasonable approach, since the mean free paths of ion collisions with atoms in low density plasma are usually long compared to the Debye length. However, Bernstein and Rabinowitz [19] in 1959 and then Goree [20] in 1992 have shown that ions can lose energy in rare collisions with atoms and become trapped in finite orbits by the electric field of a charged particle. In a steady state, the density of these trapped ions does not depend on gas pressure for rather low density of plasma. The problem of trapped ions was studied by Zobnin [21] with the help of molecular-dynamics calculations, and by Lampe [22–24] and Zagorodny [25] with the help of analytical methods. In these papers, it was shown that the density of trapped ions (with negative total energy) can be greater than the density of free ions (with positive total energy) in the vicinity of a charged dust particle, and thus plays an important role in the screening of the particle. Moreover, in collisional regimes in plasma, additional flux of trapped ions after charge exchange

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collisions with atoms can be greater than the flux of free ions. This collisional ion flux to the grain surface substantially reduces the charge of a dust grain in comparison with that predicted in OML theory. It was also demonstrated in recent papers by Hutchinson and Patacchini [27] with the help of particle-in-cell method and by Zobnin *et al.* [28] with the help of the solution of the Bhatnagar-Gross-Krook (BGK) equation for the velocity distribution of ions. These conclusions, being extremely important for dusty plasma, require careful consideration and experimental verification.

In recent papers [16,17], experimental determination of a particle charge in positive column (PC) of a glow discharge was provided for different sizes of dust particles in a wide range of neutral gas pressures (from 20 to 150 Pa). At low gas pressures ($p=20-50$ Pa) the rare collisional regimes are realized, whereas at high pressures ($p=100-150$ Pa) conditions in plasma are collisional. In all pressure ranges, the measured charges were several times smaller than those predicted in OML theory. In accordance with [17], the discrepancy between measured charges of dust particles and predictions made on the basis of OML theory was attributed merely to the effect of collisional ion flux. However, the effect of particle screening by trapped ions was not considered.

In this paper, charging and screening of a dust particle in the plasma of a glow discharge were theoretically investigated. The non-Maxwellian behavior of electron-energy distribution function (EEDF) was taken into account, which substantially modifies the electron flux to the dust grain. To take into account both the influence of collisional flux and the effect of particle screening by trapped ions, a self-consistent model for dust-particle charging was developed on the basis of the balance of the formation and destruction of trapped ions in charge exchange collisions with neutral atoms.

II. THEORETICAL MODELS

We will consider an isolated micron-sized spherical particle with radius r_0 immersed in the homogeneous low-pressure plasma of the positive column of a DC glow discharge in neon in the central part of a discharge tube. The plasma parameters used in this paper were the same as in [16,17], i.e., the constant value of electric field $E_z=2.1$ V/cm, the neon pressure range $p=20-150$ Pa, ambient plasma density is equal to $N_0=(0.9+0.03p)10^8$ cm^{-3}, and electron temperature is equal to $T_e(p)=8.3-0.02p$ eV, where p is in Pa. For these conditions the ion Debye length $\lambda_i=(T_i/4\pi e^2 N_0)^{1/2}$ is smaller than the mean free paths of ions and electrons, $l_{i,e}\sim 1/\sigma_{i,e}N_g$ ($N_g\sim 5\times 10^{15}-5\times 10^{16}$ cm^{-3} is the gas density, $\sigma_e\sim 10^{-15}$ cm^2 is the momentum cross section for electron-atom collision, and $\sigma_i=\sigma_{res}$ is the characteristic cross sections of ion-neutral atom charge exchange collisions, which has the value of 4×10^{-15} cm^2 for neon [31]). We assume that ion temperature T_i is approximately equal to gas temperature $T_g\sim 300-500$ K. Electron and ion fluxes to the particle surface charge it to a high negative value, $Z_0\sim 10^3-10^4 e$. Electric potential of charged particle surface has the value $|U_0|\sim T_e\sim 5$ eV respective to the potential of ambient plasma. The electric potential drop of external electric field E_z in the region around the charged particle is much smaller than the potential of particle, i.e., $E_z\lambda_i < E_z l_i \ll |U_0|$. Therefore, in the first approximation, we can assume that both the electric potential of a highly charged particle and radial distributions of electrons and ions around the particle are spherically symmetric ones. At the same time, the electron energy distribution function (EEDF) is formed in the ambient plasma at the conditions defined by the reduced electric field, E_z/N_g.

According to OML theory, the cross sections for electrons and single charged ions captured by the dust particle are (see, for example, [1,29,30])

$$\sigma_{cap,e}(u)=\pi r_0^2\left(1+\frac{U_0}{u}\right), \quad u>-U_0;$$
$$\sigma_{cap,i}(\varepsilon)=\pi r_0^2\left(1-\frac{U_0}{\varepsilon}\right), \tag{1}$$

where u and ε are electron and ion kinetic energies, $U_0=-e^2Z_0/r_0$ is the particle surface potential. The electron and ion fluxes to the surface of the particle are equal to

$$I_e=\sqrt{\frac{2}{m_e}}\int_{-U_0}^{\infty}\sigma_{cap,e}(u)f_e(u)u\,du, \tag{2}$$

$$I_{if}=\sqrt{\frac{2}{M_i}}\int_{0}^{\infty}\sigma_{cap,i}(\varepsilon)f_i(\varepsilon)\varepsilon\, d\varepsilon, \tag{3}$$

where $f_e(u)$ is the electron-energy distribution function, $f_i(\varepsilon)$ is the ion velocity distribution function, m_e and M_i are electron and ion masses. For Maxwellian ion distribution function, the ion flux is

$$I_{if}=\sqrt{\frac{8T_i}{\pi M_i}}\pi r_0^2 N_0\left(1-\frac{U_0}{T_i}\right). \tag{4}$$

However, in the discharge tube, the ions drift in the electric field with mean velocity, $\vec{V}_i$. The ion distribution function can be approximated by the shifted Maxwell distribution,

$$f_i(V)=n_i\left(\frac{M_i}{2\pi T_i}\right)^{3/2}\exp\left(-\frac{(\vec{V}-\vec{V}_i)^2}{2T_i/M_i}\right). \tag{5}$$

In this case, the ion flux can be easily obtained [1],

$$I_{if}=\pi r_0^2 N_0\sqrt{\frac{2T_i}{\pi M_i}}\left\{\exp\left(-\frac{M_iV_i^2}{2T_i}\right)+\frac{1+\frac{M_iV_i^2}{T_i}-2\frac{U_0}{T_i}}{\sqrt{M_iV_i^2/T_i}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{\frac{M_iV_i^2}{2T_i}}\right)\right\}. \tag{6}$$

In neon at room temperature $T_i=300$ K under reduced electric field $E_z/p>4$ V/(cm Torr), the neon ion drift velocity exceeds the mean thermal velocity, and expression (6) should be used instead of Eq. (4).

A. The influence of electron-energy distribution function on dust-particle charge

It should be noted that practically in all the papers devoted to determining a particle charge, the electron energy distribution function was assumed to be equilibrium, i.e., a Maxwellian distribution function. However, in gas discharge plasma, the EEDF is usually strongly non-Maxwellian. Only in several papers aimed at determining the dust-particle charge, the nonequilibrium EEDF was found on the basis of the solution of the Boltzmann equation [32–34]. In this paper, in order to obtain the electron distribution function, $F(\vec{v})$, in a steady state homogeneous DC glow discharge with the given value of axial electric field E_z, the kinetic Boltzmann equation was used,

$$-\frac{e}{m_e}E_z\frac{\partial F}{\partial v_z} = S^{el}(F) + \sum_k S_k^{in}(F), \tag{7}$$

where S^{el} is elastic collision integral, S_k^{in} is inelastic collision integral (includes several inelastic processes), m_e is the mass of the electron. For a weak electric field directed at axis z the assumption of low anisotropy relative to direction z is quite appropriate. The first two terms of the expansion in Legendre polynomials were taken into account,

$$F(v,v_z/v) = f_0(v) + f_1(v)\frac{v_z}{v}. \tag{8}$$

Here f_0 is the isotropic part and f_1 is the anisotropic part of EEDF. In the ambient plasma, the isotropic part f_0 is a hundred times greater than the anisotropic part f_1. Using expansion (8) in Eq. (7) and integrating over angles one can obtain the Boltzmann equation for the isotropic part of EEDF,

$$-(eE_z)^2\frac{\partial}{\partial u}\left[\frac{u}{3H(u)}\frac{\partial f_0}{\partial u}\right] = \frac{\partial}{\partial u}\left[2\frac{m_e}{M}u^2N_gQ^{el}(u)f_0\right] - \sum_k uN_gQ_k^{in}(u)f_0 + \sum_k (u+u_k^{in}) \times N_gQ_k^{in}(u+u_k)f_0(u+u_k^{in},z), \tag{9}$$

where $M=M_i$ is neutral particle mass, $Q^{el}(u)$ is the cross section for elastic processes, $Q_k{}^{in}(u)$ is the cross section for the excitation of the kth atomic state by electron impact, u_k is the energy threshold of the kth atomic state excitation, and $H(u)=N_gQ^{el}(u)+\Sigma_k N_g Q_k{}^{in}(u)$. Here, the isotropic scattering in inelastic collisions and arbitrary scattering in elastic collisions were assumed. The last term in Eq. (9) with argument $u+u_k$ describes the appearance of an electron with energy u due to electron energy loss u_k after kth inelastic process. The ionizing collisions are assumed only as electron-energy losses.

In Eq. (9), the EEDF is determined by electron-energy gain in the electric field and by electron-energy losses in elastic and inelastic collisions. Equation (9) was calculated by the iterative method starting with some initial electron-energy distribution. The double sweep method was used at each iteration until the convergence of solution.

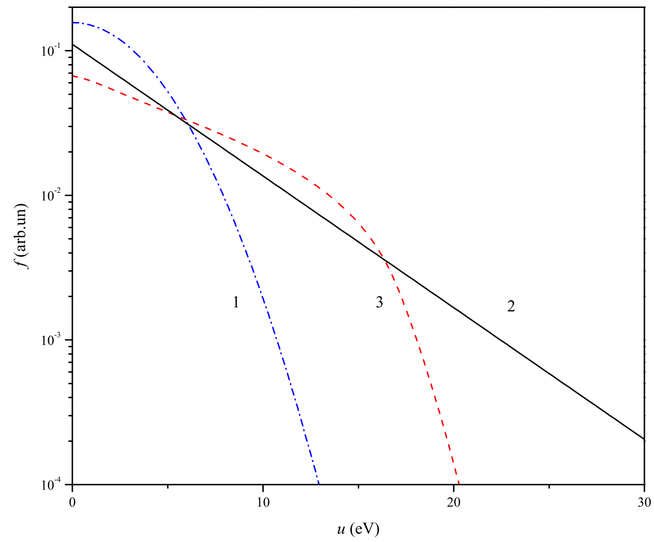

FIG. 1. (Color online) Electron-energy distribution functions: Druyvesteyn (dashed dotted line), Maxwellian (dashed line), and solution of Boltzmann equation (solid line). E=2.1 V/cm, p=133 Pa, T_e=4.77 eV.

To reveal the influence of EEDF on the dust particle charging we used three types of EEDF: Druyvesteyn, Maxwellian, and nonequilibrium EEDF obtained from the solution of the Boltzmann equation. In Fig. 1, different types of EEDF normalized to the unity are presented for neon pressure p=133 Pa and electron temperature T_e=4.77 eV: (1) the Druyvesteyn distribution $f_0(u)\sim\exp(-u^2/T_e{}^2)$, which is often used for the interpretation of probe measurements; (2) the Maxwellian distribution $f_0(u)\sim\exp(-u/T_e)$; (3) the solution of the Boltzmann equation. In Fig. 2, the solution of the Boltzmann equation for EEDF is presented for spatially uniform low-pressure DC glow discharge in neon for different neon pressures, p=20, 50, and 150 Pa. It should be noted that in the bulk plasma for the given kind of gas EEDF

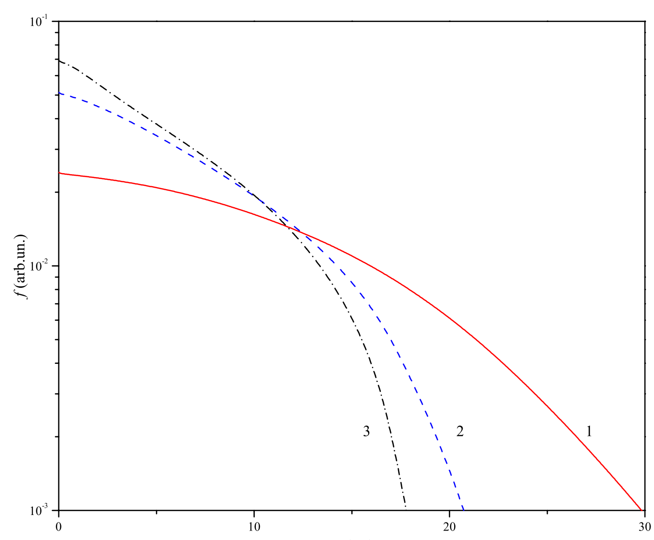

FIG. 2. (Color online) EEDF obtained from the Boltzmann equation for different gas pressures: 1–10 Pa, 2–50 Pa, 3–150 Pa in uniform electric field E=2.1 V/cm in neon.

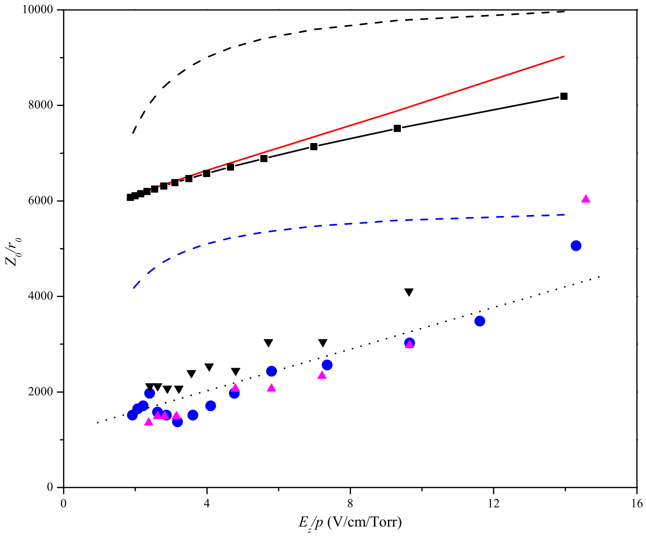

FIG. 3. (Color online) The calculated reduced particle charge number Z_0/r_0 dependence on reduced electric field E_z/p for different EEDFs: Maxwellian (dashed line), Druyvesteyn (dashed-dotted line), EEDF obtained from the Boltzmann equation [solid line, ion flux (6), -■-■-■-ion flux (4)]; symbols represent experimental data from [17] for dust particles with different radii (r_0=0.6, 1.0, and 1.3 μm), the common fit to all experimental data (dotted line).

depends on parameter E_z/N_g only. It is seen that $f_0(u)$ has almost Maxwellian behavior in the low-energy region. However, for electron energies higher than the energy of the lowest electronic state of neon, $f_0(u)$ becomes depleted due to inelastic collisions. The more gas pressure the more EEDF is depleted. At the same time, in the process of charging, a dust particle acquires a high negative charge and repulses low-energy electrons. Only electrons with energy higher than the potential of a dust particle (i.e., from the tail of EEDF) can reach its surface.

In Fig. 3, the calculated results and the experimental data from [17] are presented in the following coordinates: the particle charge number density normalized to its size, Z_0/r_0, and reduced electric field, E_z/p. The experimental data normalized to corresponding particle radius lie on the same curve. For nonequilibrium EEDF obtained with the help of the Boltzmann equation, the reduced particle charge number dependence, $Z_0(E_z/p)/r_0$, was calculated for two types of ion distribution function: for Maxwellian ion distribution function (4) and for shifted Maxwellian ion distribution function (6). It is seen that at high values of a reduced electric field E_z/p the ion drift leads to an increase of particle charge. This dependence in the bulk plasma is almost linear and has the same slope as the common fit to the experimental data.

It is seen that the calculated particle charge number depends greatly on the type of EEDF. For all three types of EEDF, the charge obtained from OML approximation exceeds the experimental data three to four times [17]. The difference between the calculated and measured charge can be explained by the presence of trapped ions around the dust particle.

B. Self-consistent model for trapped ions

Trapped ions provide a screening of the particle charge and lead to additional ion flux to the particle surface after their collisions with thermal atoms. This collisional ion flux decreases the proper dust-particle charge. In a steady state, the dust-particle charge, radial distributions of the volume charge and electric potential satisfy each other. Consequently, the dust-particle charging and screening should be considered in a self-consistent way.

We have developed a self-consistent model [35], which consists of the following submodels interconnected in an iterative procedure:

(1) Determination of radial distribution of trapped ions, free ions, and electrons in a given electric potential and for a given charge of the particle, Z_0.

(2) Determination of collisional ion flux on the grain surface, and recalculation of particle charge, Z_0, taking into account the found fluxes of free and trapped ions and electrons.

(3) Determination of electric potential radial distribution, $U(r)$, with the help of the Poisson equation for obtained distribution of ions and electrons.

Radial distribution of free ions, $N_{if}(r)$, has the form [23,25]

$$N_{if}(r) = \frac{N_0}{2}\frac{2}{\sqrt{\pi}}\frac{1}{T_i^{3/2}}\left\{\int_{E_m(r)}^{\infty} d\varepsilon \exp(-\varepsilon/T_i)\left[\sqrt{\varepsilon - U(r)} + \sqrt{1-\frac{r_0^2}{r^2}}\sqrt{\varepsilon - E_0(r)}\right]\right\}, \tag{10}$$

where N_0 is the density of ambient plasma, ε is ion kinetic energy, T_i is ion temperature, and

$$E_0(r) = \frac{r^2U(r) - {r_0}^2U(r_0)}{r^2 - r_0^2}. \tag{11}$$

The lower limit of integration, $E_m(r)$, is determined from the condition of a positive value of the second radical in Eq. (10), that is,

$$\begin{cases} E_m(r) = 0, & \text{if } E_0(r) \leqslant 0, \\ E_m(r) = \dfrac{r^2U(r) - {r_0}^2U(r_0)}{r^2 - r_0^2} = E_0(r), & \text{if } E_0(r) > 0. \end{cases} \tag{12}$$

Let us estimate the number density of trapped ions N_{tr} and compare it with the number density of free ions N_{if}. First of all we determine the conditions of ion trapping in a bound orbit as a result of the resonant charge exchange collision with a thermal atom at distance R from the charged particle. After a collision, the atom is transformed into an ion with velocity $\vec{v}$ characterized by angle θ to the direction of radius-vector $\vec{R}$. The first condition requires a forming ion to have negative energy $E_t(R)$,

$$E_t(R) = \varepsilon + U(R) < 0. \tag{13}$$

The second condition is that a trapped ion cannot reach the particle surface, i.e., the minimal distance from the trapped ion to the particle center $r_{\min}$ satisfies the relation

$$r_{\min} \geqslant r_0, \tag{14}$$

where r_0 is the particle radius. The orbital momentum of an ion after the charge exchange process is $L = MRv \sin\theta$, and the total ion energy $E_t(r)$ at distance r from the particle center is [36]

$$E_t(r) = E_t(R) = \frac{Mv_r^2}{2} + \frac{L^2}{2Mr^2} + U(r) = \frac{Mv_r^2}{2} + \varepsilon \frac{R^2 \sin^2\theta}{r^2} + U(r) < 0, \tag{15}$$

where $\varepsilon = Mv^2/2$ is the ion energy at the moment of charge exchange, M is the ion mass, v_r is the radial ion velocity. Let us introduce a minimum ion distance from the particle r_{min} on the basis of relation $v_r(r_{min}) = 0$. Condition $r_{min} = r_0$ gives the range of angles $\theta_{min} < \theta < \pi - \theta_{min}$ of ion velocity after the charge exchange collision with an atom in which the ion becomes trapped,

$$\sin\theta_{\min}(R,\varepsilon) = \frac{r_0}{R}\sqrt{1 + \frac{U(R) - U(r_0)}{\varepsilon}}. \tag{16}$$

Ion energy should satisfy the condition

$$r_0^2 \frac{U(R) - U(r_0)}{R^2 - r_0^2} < \varepsilon < -U(R). \tag{17}$$

We now determine probability $P_{tr}(R,\varepsilon)$ that an ion that results from the resonant charge exchange process with a thermal atom would be trapped,

$$P_{tr}(R,\varepsilon) = \frac{1}{4\pi}\int_{\Omega_{tr}} \sin\theta d\theta d\varphi = \frac{1}{2}\int_{\theta_{\min}}^{\pi-\theta_{\min}} \sin\theta d\theta = \cos\theta_{\min}(R,\varepsilon). \tag{18}$$

Taking the Maxwell distribution function of atoms on energies

$$f(\varepsilon) = \frac{2}{\sqrt{\pi}T_i^{3/2}} \exp\left(-\frac{\varepsilon}{T_i}\right)\varepsilon^{1/2}$$

that is normalized to unity ($\int f(\varepsilon)d\varepsilon = 1$), for the average probability we find that a formed ion is trapped,

$$P_{tr}(r) = \langle \cos\theta_{\min}(r,\varepsilon)\rangle = \sqrt{1 - \frac{r_0^2}{r^2}}\frac{2}{\sqrt{\pi}}\int_{y_{\min}}^{y_{\max}} \sqrt{y - \frac{E_1(r)}{T_i}} e^{-y} dy, \tag{19}$$

where $y = \varepsilon/T_i$, $E_1(r) = \frac{r_0^2[U(r) - U(r_0)]}{(r^2 - r_0^2)}$. The upper limit in the integral

$$y_{\max} = -U(r)/T_i \tag{20}$$

is determined from Eq. (13), which is the condition of ions becoming trapped, and

$$y_{\min}(r) = E_1(r)/T_i = \frac{r_0^2[U(r) - U(r_0)]}{(r^2 - r_0^2)T_i} > 0. \tag{21}$$

From Eq. (19), it can be concluded that trapped ions can be formed in the region from r_0 to R_0 only, where R_0 is defined from the relation

$$r_0^2 U(r_0) = R_0^2 U(R_0). \tag{22}$$

In the same way, we introduce probability $P_{fall}(r)$ that after a charge exchange collision an ion falls on the particle, and probability $P_{free}(r)$ that the ion acquires positive energy and therefore runs away to infinity (or falls on the particle for some interval of angles, θ). These probabilities are equal to

$$P_{fall}(r) = \frac{2}{\sqrt{\pi}}\int_0^{y_{\max}} dy e^{-y}\sqrt{y} - \sqrt{1 - \frac{r_0^2}{r^2}}\frac{2}{\sqrt{\pi}}\int_{y_{\min}}^{y_{\max}} dy e^{-y}\sqrt{y - y_{\min}(r)}, \tag{23}$$

where we take into account the fact that ions with small energy $[\varepsilon > E_1(r)]$ fall on the particle irrespective of their angular momentum, and

$$P_{free}(r) = \frac{2}{\sqrt{\pi}}\int_{y_{\max}}^{\infty} \sqrt{y}\exp(-y)dy. \tag{24}$$

It can be easily verified that

$$P_{tr}(r) + P_{free}(r) + P_{fall}(r) = 1. \tag{25}$$

Now we can introduce a balance equation for the number density of trapped ions. In unit time as a result of charge exchange collisions of free and trapped ions with neutral atoms with collisional frequency ν, new trapped ions are formed at the point R in spherical layer $4\pi R^2 dR$ with the rate equal to $4\pi R^2 dR\, N_{if}(r)\nu$ (here we neglect the velocity dependence of ν as it was done in the paper [23]). The newly born ions have energy ε_a and angle of movement θ_a. Some of them have negative total energy, $E_t = \varepsilon_a + U(R) < 0$, and thus they are trapped, and move around a charged particle contributing to the density of trapped ions at different points along their finite trajectories. In layer $4\pi r^2 dr$, this input is proportional to the ratio of residence time in dr, $dt(R,r,\varepsilon_a,\theta_a) = dr/v_r(R,r,\varepsilon_a,\theta_a)$, to the time $T(R,\varepsilon_a,\theta_a)$ of an ion moving from $r_{min}(R,\varepsilon_a,\theta_a)$ to $r_{max}(R,\varepsilon_a,\theta_a)$. Here, $v_r(R,r,\varepsilon_a,\theta_a)$ is radial velocity of the trapped ion formed at point R and transferred to point r,

$$v_r(R,r,\varepsilon_a,\theta_a) = \pm\sqrt{\frac{2\varepsilon_a}{M}}\sqrt{\left(1 - \frac{R^2}{r^2}\sin^2\theta_a + \frac{U(R) - U(r)}{\varepsilon_a}\right)}. \tag{26}$$

In this expression, zeros give the turning points of ion, $r_{min}(R,\varepsilon_a,\theta_a)$ and $r_{max}(R,\varepsilon_a,\theta_a)$. Half-period of ion movement along a trajectory is [36]

$$T(R,\varepsilon_a,\theta_a) = \sqrt{\frac{M}{2\varepsilon_a}}\int_{r_{\min}}^{r_{\max}} \frac{dr}{\sqrt{\left(1-\frac{R^2}{r^2}\sin^2\theta_a+\frac{U(R)-U(r)}{\varepsilon_a}\right)}}. \quad (27)$$

Integrating over the whole region around a particle where trapped ions can be formed (from r_0 to R_0) and averaging over kinetic energy ε_a and angles θ_a, we can receive the rate of trapped ions creation. This rate is equal to the rate of trapped ions loss in the layer $4\pi r^2 dr$. As a result we can receive a balance equation for trapped ions,

$$\nu N_{tr}(r) = \nu\int_{r_0}^{R_0} dR\frac{R^2}{r^2}[N_{if}(R)+N_{tr}(R)]\int_{E_1(R)}^{-U(R)} d\varepsilon_a \frac{2}{\sqrt{\pi}}\frac{\sqrt{\varepsilon_a}}{T_i^{3/2}} \times\exp\left(-\frac{\varepsilon_a}{T_i}\right)\int_{\pi-\theta_m}^{\theta_m} d\theta_a\frac{\sin\theta_a}{2} \times\frac{\Theta\left(1-\frac{R^2}{r^2}\sin^2\theta_a+\frac{U(R)-U(r)}{\varepsilon_a}\right)}{\sqrt{1-\frac{R^2}{r^2}\sin^2\theta_a+\frac{U(R)-U(r)}{\varepsilon_a}}} \times\frac{1}{\int_{r_{\min}(R,\varepsilon_a,\theta_a)}^{r_{\max}(R,\varepsilon_a,\theta_a)}\frac{dr'}{\sqrt{1-\frac{R^2}{r'^2}\sin^2\theta_a+\frac{U(R)-U(r')}{\varepsilon_a}}}}. \quad (28)$$

In this equation, we introduce the Heaviside step function $[\Theta(x)=1,\ x>0;\ \Theta(x)=0,\ x<0]$, which ensures the calculation of integrals only in the accessible region of parameters. Equation (28) is valid for rare collisional conditions in plasma in the first approximation. In the collisional case, the Vlasov-Boltzmann equation or the Bhatnagar-Gross-Krook (BGK) equation for the velocity distribution of ions should be solved (see the recent paper [28]). The collisional case was also studied with the help of particle-in-cell calculations by Hutchinson and Patacchini [27]. It should be stressed that the model presented by Eq. (28) is consistent with the low-frequency limit of the solution of BGK equation obtained in [28].

Multiplying Eq. (28) by $4\pi r^2 dr$ and integrating over r from r_0 to R_0, we can receive the total balance of trapped ions creation and loss in the whole region around a charged particle,

$$4\pi\int_{r_0}^{R_0} dRR^2\nu N_{tr}(R)[P_{fall}(R)+P_{free}(R)] = 4\pi\int_{r_0}^{R_0} dRR^2\nu N_{if}(R)P_{tr}(R). \quad (29)$$

Here, we take into account the relation (25).

The calculation of Eq. (28) is rather difficult due to the singularities of the integral kernel in right side of Eq. (28) and due to the necessity of calculating the self-consistent potential $U(r)$ that require an iterative approach. The analysis of this equation will be presented in a subsequent paper [35]. In this paper we consider a simplified version of Eq. (28). It is seen that singularities of the kernel in Eq. (28) are most pronounced at $R=r$ and $\theta_a=\pi/2$. It means that a trapped ion on its trajectory spends most time near the remote turning point. It is reasonable to substitute the arguments of the function $R^2N_{if}(R)$ in the right side of Eq. (28) for their values at point r. After integration over R we can receive an approximate form of the balance equation

$$N_g\overline{\sigma_{res}\mathrm{v}_{if}(r)}N_{if}(r)P_{tr}(r) = N_g\overline{\sigma_{res}\mathrm{v}_{tr}(r)}N_{tr}(r)[P_{fall}(r) + P_{free}(r)], \quad (30)$$

where instead of frequency ν we introduce $N_g\overline{\sigma_{res}\mathrm{v}_{if}(r)}$, and $\overline{\mathrm{v}_{if}(r)}$ is an averaged relative velocity between a free ion and atom, $\overline{\mathrm{v}_{tr}(r)}$ is an averaged relative velocity between a trapped ion and atom.

It is clear that our approximation leads to a slightly narrower radial distribution of trapped ions than the exact distribution. From Eq. (29), it is seen that approximation (30) is in full agreement with the total balance for trapped ions. From the balance equation (28), it follows that the number density of trapped (bound) ions does not depend on the atom number density [20] and on the cross section of charge exchange, σ_{res}, because the formation and destruction of bound ions is determined by the same process, and the cross section of resonant charge exchange is independent on the ion-atom relative velocity [20,31].

In formula (30), let us assume the ratio of averaged velocities of free and bound ions to be

$$\varsigma(r) = \frac{\overline{\sigma_{res}\mathrm{v}_{if}(r)}}{\overline{\sigma_{res}\mathrm{v}_{tr}(r)}}, \quad (31)$$

we will find that the number density of trapped ions is equal to

$$N_{tr}(r) = N_{if}(r)\varsigma(r)\frac{P_{tr}(r)}{[P_{fall}(r)+P_{free}(r)]}. \quad (32)$$

This expression agrees within the denominator in Eq. (32) with the result obtained in the paper by Bystrenko and Zagorodny [25]. Possibly, this approximation leads to a narrower radial distribution of trapped ions than the exact distribution obtained from Eq. (28). However, as it is seen in Eq. (29), it does not change the total number of trapped ions.

It should be stressed that the balance equations [(28) and (30)] for trapped and free ions are valid only on average. Instead of Eq. (28), the Vlasov-Boltzmann equation for the velocity distribution of ions should be solved. The division of ions into two groups (free and trapped ions) can be justified only for a low density conditions in plasma. The ratio $\varsigma(r)$ depends on the model chosen for ion-atom charge exchange collision. For a constant frequency of charge exchange collision, this ratio is equal to unity identically. For velocity independent cross section, this ratio can be also approximated by $\varsigma(r)=v_{if}(r)/v_{tr}(r)\approx 1$. This problem will be considered in more details in subsequent paper [35].

For the determination of electric potential, it is necessary to consider the Poisson equation

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU(r)}{dr}\right) = -\frac{1}{r}\frac{d^2}{dr^2}[rU(r)] = 4\pi e^2\{N_{if}(r) + N_{tr}(r) - N_e(r)\}. \quad (33)$$

The solution can be presented in the form [37]

$$U(r) = -\frac{e^2 Z_0}{r} + e^2\int_{r_0}^{\infty} dx x^2 \int_0^{\pi} d\vartheta' \sin\vartheta' \int_0^{2\pi} d\varphi' \times \frac{\Delta N(x)}{\sqrt{r^2 - 2rx\cos\Theta + x^2}}, \quad (34)$$

where $\Delta N(r)=N_{if}(r)+N_{tr}(r)-N_e(r)$ is the density of volume charge of ions and electrons at the point r, $\cos\Theta = \cos\vartheta\cos\vartheta' + \sin\vartheta\sin\vartheta'\cos(\varphi-\varphi')$.If we expand the denominator of Eq. (34) in Legandre polynomials and provide integration over angles taking into account symmetry of volume charge distribution, we can obtain

$$U(r) = -\frac{e^2 Z_0}{r} + \frac{4\pi e^2}{r}\int_{r_0}^{r} dx x^2 \Delta N(x) + 4\pi e^2 \int_r^{\infty} dx x \Delta N(x). \quad (35)$$

The electric field radial distribution is equal to

$$eE(r) = -\frac{e^2 Z_0}{r^2} + \frac{e^2}{r^2}\int_{r_0}^{r} 4\pi \Delta N(x) x^2 dx, \quad (36)$$

that agrees with the Gauss theorem, since the integral is equal to the charge of ions (free and trapped) and electrons in the volume between the particle and the sphere of radius r,

$$Q(r) = 4\pi e\int_{r_0}^{r} \Delta N(r) r^2 dr. \quad (37)$$

The trapped ions charge is equal to

$$Q_{tr}(r) = 4\pi e\int_{r_0}^{r} N_{tr}(r) r^2 dr. \quad (38)$$

From Eq. (35) it is seen that the radial distribution of electric potential $U(r)$ has a finite jump from the value $U_0 = -eZ_0/r_0$ to $U(r_0) = U_0 + 4\pi e^2 \int_{r_o}^{\infty} dr r \Delta N(r)$. This fact is important for the calculation of self-consistent distributions of electric potential and ion densities.

The calculation procedure was the following. At the initial step the dust-particle charge number Z_0 was chosen for the given discharge parameters (gas and ambient plasma densities, N_g, N_0, electric field, E_z, ion temperature, T_i) according to the OML model. Assuming that the initial electric potential radial distribution is the Debye-Hückel one,

$$U(r) = -\frac{e^2 Z_0}{r}\exp\left(-\frac{r}{\lambda_i}\right), \quad (39)$$

we calculated the radial distributions of all probabilities [Eqs. (19), (23), and (24)] and ion number densities $N_{if}(r)$, $N_{tr}(r)$. Then a distribution of self-consistent electric potential $U(r)$ was found with the help of the Poisson equation. The final electric potential and volume charge distributions as well as collisional ion flux to the particle surface, I_{tr}, were found using the iterative method. The collisional ion flux of trapped ions was calculated with the help of formula

$$I_{tr} = 4\pi\int_{r_0}^{\infty} P_{fall}(r) N_{tr}(r) N_g \sigma_{res} \overline{v_{tr}(r)} r^2 dr, \quad (40)$$

which is valid under low density plasma conditions in the first approximation.

The total ion flux to the particle is the sum of free ion flux (OML model) and collisional ion flux, $I_i = I_{if} + I_{tr}$. Equating total ion flux to the electron flux, I_e, we can find a new dust-particle charge. For this value of particle charge, the above described iterative procedure was repeated. It should be stressed that the final values of dust-particle charge, the distributions of charged particles and electric potential do not depend neither on the choice of initial value of Z_0 nor on the initial electric potential distribution $U(r)$.

Below, the calculated results are presented for neon pressure $p=100$ Pa and dust-particle radius $r_0 = 1$ μm. For this condition, the Debye length is equal to $\lambda_i = 65 r_0$. In Fig. 4, it can be seen that the probability of ion trapping P_{tr} is equal to zero near the particle and is maximum at some distance of about $0.5\lambda_i$ from the particle, then it tends to zero. The probability $P_{fall}(r)$ increases from zero to unity with the decrease of distance r to the particle due to the increase of particle attraction potential.

In Fig. 5, the radial distributions of trapped ions N_{tr}, free ions N_{if}, and electrons N_e, as well as the total volume charge ΔN, are presented. All the densities are normalized to the ambient plasma density N_0. It is seen that free ions and electron densities become equal to each other at the distances of about several Debye lengths from the particle. The trapped ion density $N_{tr}(r)$ and the probability $P_{tr}(r)$ have maximums

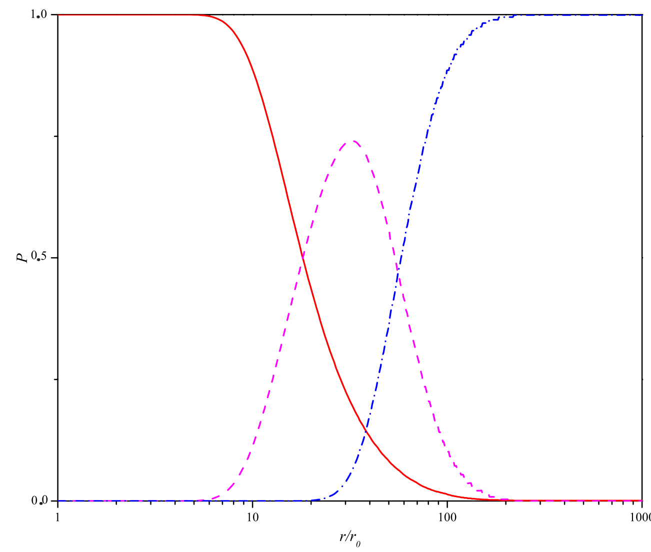

FIG. 4. (Color online) Radial distribution of probabilities: P_{fall} (solid line), P_{tr} (dashed line), and P_{free} (dashed-dotted line). $\lambda_i = 65 r_0$.

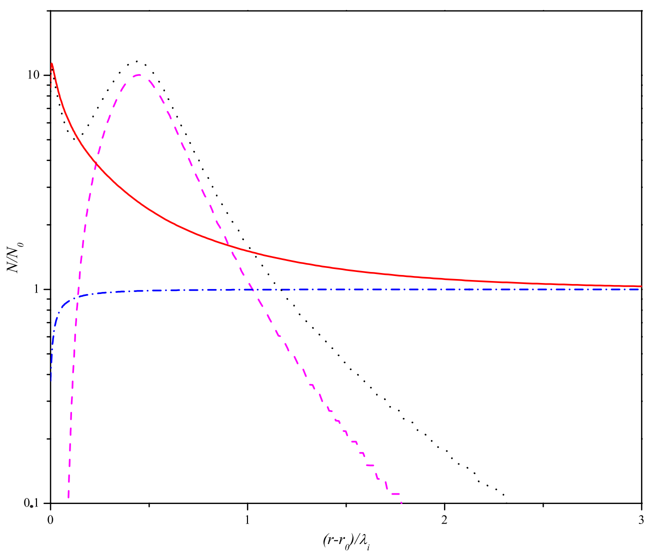

FIG. 5. (Color online) Radial distribution of free ions $N_{if}(r)$ (solid line), trapped ions $N_{tr}(r)$ (dashed lines), electrons $N_e(r)$ (dashed-dotted line), and total volume charge $\Delta N(r)$ (dotted line).

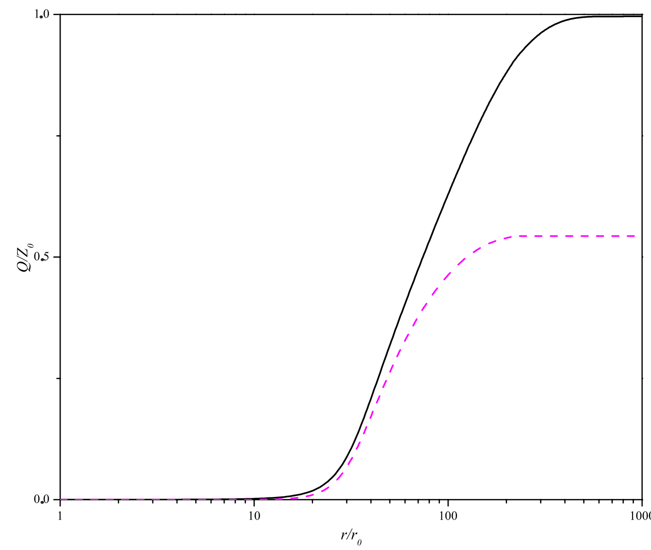

FIG. 6. (Color online) Radial distribution of total volume charge $Q(r)$ (solid line) and the trapped ions charge $Q_{tr}(r)$ (dashed line) normalized to dust-particle charge Z_0.

approximately at the same radial positions in agreement with the results of [25]. For distances beyond this maximum the behavior of $N_{tr}(r)$ is in full agreement with the results of [23] and calculations of [27,28] for $\nu \to 0$. At small distances, $N_{tr}(r)$ decreases [possibly due to the approximation made in Eq. (30)], which contradicts the results of [23]. However, this fact does not influence the value of the total charge of trapped ions $Q_{tr}(r\to\infty)$, which is equal to 40–60 % (as in [23]) of the proper charge of the dust particle $|eZ_0|$ depending on λ_i. This is the consequence of the total balance of trapped ions in the whole region around a charged particle, Eq. (29). It should be mentioned again that we assume that the orbital period of a trapped ion is short compared to the collision time. We also assume that ions that fall onto a particle immediately disappear from the trapped ion distribution. In collisional regimes such ions can give considerable contribution to the ion density, see [27,28].

As it follows from the results of the self-consistent model, the total volume charge is equal to Z_0, i.e., $Q(r\to\infty)/eZ_0 \to 1$ (see Fig. 6). For $\lambda_i/r_0=65$ and $p=100$ Pa, the total trapped ions charge is equal to $Q_{tr}(r\to\infty)=0.54\ eZ_0$, $Z_0=5000$. It means that 54% of the total screening of particle charge $-eZ_0$ is provided by trapped ions charge Q_{tr}. It should be noted that the presented iterative procedure leads to a full screening of a dust-particle charge Z_0 of a volume charge Q self-consistently for any conditions.

Finally, we carried out calculations for different gas pressures $p=20-150$ Pa according to experimental data [17]. In Fig. 7, the following dependencies on gas pressure can be seen: (1) the particle charge number $Z_{OML}(p)$ obtained with the help of OML model and the EEDF received from the Boltzmann equation; (2) the particle charge number $Z_0(p)$ obtained with the help of the self-consistent model; (3) experimental results [17]; (4) the effective particle charge number, $Z_{eff}=Z_0-Q_{tr}/e$. It is seen that the effective charge number Z_{eff} dependence on gas pressure presented in Fig. 7 is in fairly good agreement with the experimental data obtained in [17], especially in the low pressure region. It should be stressed that for experimental data [17] at low gas pressures (20–50 Pa) the conditions in plasma can be regarded as low collisional ones. Under such conditions the collisional ion flux is not essential and does not lead to a substantial reduction of a particle charge (in agreement with the prediction made in [23,27,28]). The total number of trapped ions is ~50% of the proper charge of the dust particle, and these trapped ions take part in the shielding of the charged particle. Under higher pressures ($p=100-150$ Pa), the plasma conditions are collisional. In this pressure region, trapped ions with negative total energy take part in consecutive collisions

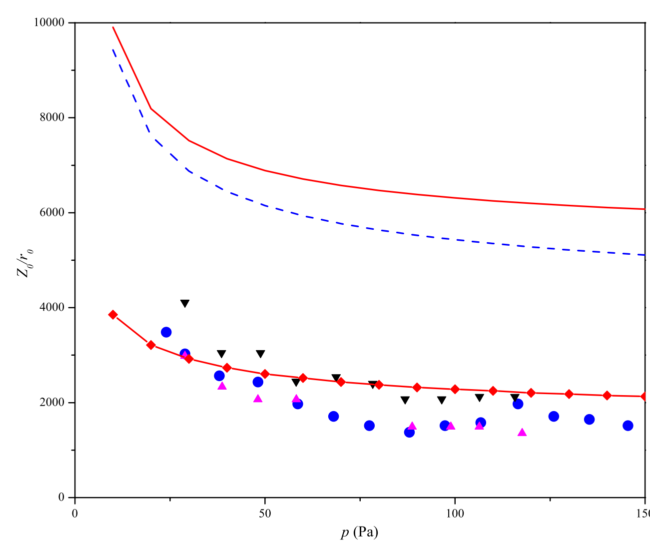

FIG. 7. (Color online) Dependence of particle charge Z_0 normalized to particle radius r_0 on gas pressure: result of OML model, $Z_{OML}(p)$ (solid line); self-consistent solution for $Z_0(p)$ (dashed line); dust-particle effective charge $Z_{eff}(p)$ (-◆-◆-◆-); symbols are experimental data [17] for the particles with different radius (r_0=0.6, 1.0, and 1.3 μm).

that lead to their approaching the surface of the particle rather than to their orbiting movement around the particle. The influence of collisional ion flux is increased with the increase of frequency ν, and the influence of trapped ions that take the form of a bound ionic coat is decreased. This case should be treated with the help of the solution of BGK equation [28]. However, in this paper we extrapolated our results into the collisional regime.

III. CONCLUSION

We have found out that calculated dust-particle charge number strongly depends on the choice of EEDF. For EEDF obtained from the kinetic Boltzmann equation, the calculated charge-pressure dependence and experimental data have the same profiles in the whole investigated region. The non-Maxwellian character of the electron distribution function in plasma of a glow discharge leads to a decrease of the particle charge in comparison to that predicted by OML theory. However, the absolute values of an estimated charge exceed the measured values [17] almost two to three times. To understand such a discrepancy, the effects of trapped ion screening and collisional flux were considered. In weakly collisional regime in the plasma, the decrease of the dust-particle charge connected with the collisional flux is not essential. For these regimes (low gas pressures), it is more important that trapped ions screen the proper charge of a particle. A great number of trapped ions around a charged dust particle are in a bound state. The electrostatic force acting on the particle with an ionic coat is proportional to electric field strength and the effective charge of the particle, Z_{eff}, which is equal to the difference between the particle charge and the ionic coat charge, $Z_{eff}=Z_0-Z_{tr}$. In the experimental measurements based on the electrostatic interaction between two particles [15] or between the particle and an electric field [17], the effective charge can be obtained. The proper dust-particle charge Z_0 cannot be even estimated without the knowledge of trapped ions charge number, Z_{tr}.

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